EVALUATION OF TRANSMISSION LOSS ALLOCATION IN Deregulated Power System by Using Different Methods

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ABSTRACT - In this paper different Transmission Loss Allocation methodology are considered. With the increase of the competition level in electricity markets the problem of Transmission Loss Allocation (TLA) among the power system agents has become more important. Which is not available in the literature, has been made is the comparison between them. The methods considered in this paper are; Postage Stamp (PS), Proportional Sharing Principle (PSP) and Loss Function Decomposition (LFD) methods. Algorithms have been implemented for these methods. These methodologies are illustrated on IEEE 5-bus system and IEEE-14 bus system. The theoretical values are verified with the simulation results obtained from the MATLAB programming.

Keywords: Postage Stamp Method, Proportional Sharing Principle Method, Loss Function Decomposition Method, Transmission Loss Allocation.

I. INTRODUCTION

Generators submit hourly energy bids and their corresponding prices to the power exchange (PX), while consumers submit hourly energy demands and their respective maximum buying prices. The PX market operator, on an hourly basis, builds the generator increasing stepwise curve of bids and the consumer decreasing stepwise curve of demands. The crossing of these two curves determines the hourly market-clearing price and allows determining how much energy each generator is allocated to produce. Hourly auctions are usually performed one day ahead. The economic background of the electric power utility sector is changing; all over the world, the power system is under restructuring. The main objective of this restructured power system is to create the competition in the electricity market. One of the key issues of this restructured power system is Transmission Loss Allocation. The transmission loss allocation can ensure an efficient utilisation of the existing network, its security and be a device to send correct signals concerning the development of new generating capacities. Transmission loss allocation is a subject of considerable debate among various customers in the electricity industry. From the viewpoint of state regulators and load interests, loss allocation raises questions of increased or reduced electricity rates for end-use customers. This paper describes a variety of transmission loss allocation methodologies and evaluates their respective properties. A description of transmission planning, including how studies are conducted, is essential to understanding the kinds of benefits transmission planning identifies and places loss allocation methodologies into context. In this paper, three different loss allocation methods are discussed; Postage Stamp method (PS), Proportional Sharing Principle (PSP) method and Loss Function Decomposition (LFD) method. The Postage Stamp method does not take into account the network and produces substantially different results than other methods. The proportional sharing procedure takes into account the network. The Loss Function Decomposition method provides fair, efficient and accurate loss allocation to the transmission system customers. In this paper, the applications of PS, PSP and LFD methods are illustrated by using sample 5-Bus system and IEEE 14-Bus test system to perform the loss allocation to the transmission customers.

II. POSTAGE STAMP METHOD

Postage stamp methodology is the simplest and easy to implement methodology of transmission pricing. A postage stamp rate is a fixed charge per unit of power transmitted within a particular zone. The rate does not take into account the distance involved in the wheeling. There are various versions of postage stamp methodology. In some versions, both, generators and loads are charged for transmission usage, while in others, only loads pay for the same. Some variants charge loads for their
A. Transmission Loss Allocation

Based on main assumption the PS method proportionally allocates 50% of losses to the generator and 50% to the demands, that is

\[
L_{PGi} = \frac{L_{PGi}}{2} P_G
\]

\[
L_{PDj} = \frac{L_{PDj}}{2} P_D
\]

where,

PGi, PDj - Real Power Generation and Load at buses i and j

PG - Total Power Generation and Load of the System

L - Losses

III. PROPORTIONAL SHARING PRINCIPLE METHOD

The tracing methodology is based on the assumption that, at any network node, the incoming flows are proportionally distributed among the outgoing flows. This assumption can be neither proved nor disproved physically and the authors aim to rationalise it. The analysis presented here is of the loss allocation problem. First it is shown that the proportionality assumption leads to the cost allocation which is aggregation invariant.

PSP method, sometimes is also called flow-tracing scheme. In this method, it is assumed that nodal inflows are shared proportionally among nodal outflows. This method can deal with both dc power flow and ac power flows; i.e., it can be used to find contributions of both active and reactive power flows.

A. Formation of Branch Incidence Matrix (B)

Consider a network consisting of n nodes and m branches and define P, Q, PG, QG, PD and QD as (n 1) vectors of nodal flows, nodal real and reactive power generations and nodal demands respectively, and F and Fq as (m 1) vectors of real and reactive branch power flows. The (m n) incidence matrix B can be formed by considering +1” for sending end node and -1” for receiving end node. If there is any node, that does not belong to particular branch can be assigned zero. This matrix B can be split into matrix Bu consisting of +1”s and Bd consisting of -1”s.

B. Transmission Loss Allocation

The adjacency matrix (D) is defined as (n n) matrix with [D]ij =1 if there is a flow from node i to node j. The adjacency matrix can be expressed as follows.

\[
D = -B_d^T B_u
\]

The branch flow matrix (Fd) with (n n) size, such that its (ij) element is equal to the flow in line i-j towards node j (i.e., downstream). Fd can be expressed as follows,

\[
F_d = -B_d^T \text{diag}(F) B_u
\]

where, F is power flow vector which is taken from the Newton Raphson Load Flow method.

The element (ij) of FdT is equal to the flow in line ij towards i (i.e., upstream). The Kirchhoff Current Law (KCL) can be expressed to compute nodal powers as,

\[
P = P_G + F_d^T \cdot 1
\]

where, 1 is (n 1) vector of ones.

The upstream allocation matrix (Au) is the sparse and non-symmetric matrix and it can be expressed as,

\[
A_u = I + B_d^T \text{diag}(F) B_u \left[ \text{diag}(p) \right]^{-1}
\]

where, I is an Identity matrix. Similarly, the downstream allocation matrix (Ad) can be expressed as,

\[
A_d = I + B_d^T \text{diag}(F) B_u \left[ \text{diag}(p) \right]^{-1}
\]

The Proportional Sharing Principle method can also express branch flows as the sum of components supplied from individual generators or to individual loads.

\[
P_{i-j}(\text{gross}) = \frac{P_{ij}}{P_i} \sum_{k=1}^{n} \left[ A_u(i,k) \right]^{-1} P_{Gk} \text{ for } j \in \alpha_i^d
\]

where, id = set of nodes supplied directly from node i

Pi = Nodal power

k = Buses (Generator bus)

PG k = Generating power at bus k

PijBranch power flow (i=upstream, j \in downstream)

\[
P_{i-j}(\text{net}) = \frac{P_{ij}}{P_i} \sum_{k=1}^{n} \left[ A_d(i,k) \right]^{-1} P_{Dk} \text{ for } j \in \alpha_i^u
\]
where, \( P_{Dk} = \) load at bus \( kk = \) Buses (Load bus)
\( R_j = \) Branch power flow\((j=\)upstream, \(i=\)downstream)
\( \alpha \) Set of nodes supplying node \( i \)

In order to assign 50% of losses to the generation and 50% to the demand, the final generation and demand per bus are computed as,

\[
P_{G_i} = \frac{P_{\text{net}} + P_{G_i}}{2} \quad (10)
\]

Finally the real power losses allocated to every generator and demand are computed as,

\[
L'_{Gi} = P_{Gi} - P'_{Gi} \quad (12)
\]

\[
L_D = P_{Dj} - P'_{Dj} \quad (13)
\]

**IV. LOSS FUNCTION DECOMPOSITION METHOD**

Loss due to load currents are obtained by assuming that all generators act as ideal voltage sources with no circulating currents between them. In such a case, generation nodes are short circuited and the load nodes are considered as current sources.

Loss Function Decomposition (LFD) method is a different method for allocating transmission losses among loads and generators by using Current Projection concept for Pool based markets. This is the best method for allocating losses which in turn fix the costs among loads and generators.

In this method, first, loss function is decomposed into two components, load losses and no load losses. Therefore, by using current projection concept, share on branch current due to loads and generators are determined.

**A. Subsidized LFD Method**

i) Transmission Loss Allocation to Loads

Loss due to load currents are obtained by assuming that all generators act as ideal voltage sources with no circulating currents between them. In such a case, generation nodes are short circuited and the load nodes are considered as current sources. Considering the node equations of the power system

and by proper partitioning of \( Y_{BUS} \), the power system equations can be written as:

\[
[I_G] = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} [V_G]
\]

The current injection at load bus is represented by \( I_L \), the contribution of \( I_L \) to the generator bus voltage is zero and its contributions to the load bus voltage will be calculated as:

\[
V_k^L = Y_k^{-1}e I_L, k = 1,2,\ldots,n
\]

where, \( L \) is number of loads and \( e \) is an \( L \times 1 \) dimension vector with value of 1 at position \( L \) and all the others equal to 0. The contribution of \( I_L \) to the voltage drop across branch is computed as:

\[
\Delta V_b^L = V_{bf}^L - V_{bt}^L
\]

here, the subscripts \( bf, bt \) represent the “from” bus and the “to” bus of branch, respectively.

The contribution of \( I_L \) to the current through branch can be computed as:

\[
I_b^L = \frac{\Delta V_b^L}{Z_b}
\]

where \( Z_b = R_b + jX_b \) is the complex serial impedance of branch \( b \). Here the shunt elements of lines are not taken into account because it is only focused on active loss allocation. allocation, \( I_b^L \) is the branch current contribution due to load.

According to the superposition principle, the branch current contribution due to each current injection is as follow:

\[
I_b = \sum_{L=1}^l I_b^L
\]

Let \( Ip_bL \) denote the projection vector of \( I_bL \) in the direction of \( I_b \), which is defined to be the current projection component of branch \( b \) produced by the current injection at load \( L \) and it is calculated as:

\[
I_{pb}^L = |I_b^L| \cos(\varphi_b^L - \varphi_b) e^{j\Phi_b}
\]

where, \( \Phi_bL, \Phi_b \) are phase angles of \( I_bL, I_b \).

After obtaining the branch current contributions due to load current injections, its allocated loss of branch \( b \) is accordingly given by the following formula:
\[ L^L_b = (V_{bf} - V_b) I^L_{pb} = |I^L_b| \cos(\phi^L_b) e^{-j \phi^L_b} I^L_b Z_b = |I^L_b| |I_b| \cos(\phi^L_b - \phi_b) z_b \]

\[ p_{lossb}^L = |I^L_b| |I_b| \cos(\phi^L_b - \phi_b) r_b \]

where, \( r \) = Resistance of a branch b

According to Superposition principle the branch current contributions due to each generator current injection is calculated as:

\[ I_b = \sum_{G=1}^G I^G_b \]

In this stage, like the previous one, \( I_{pbG} \) denote the projection vector of \( I_bG \) in the direction of \( I_b \), which is defined to be the current projection component of branch b produced by the current injection at generator G and it is calculated as:

\[ I_{pbG}^G = |I^G_b| \cos(\phi^G_b - \phi_b) e^{j \phi_b} \]

Consequently, allocated loss of branch b due to mismatch in generator’s voltage derived as:

\[ p_{lossb}^G = |I^G_b| |I_b| \cos(\phi^G_b - \phi_b) r_b \]

where, \( \phi^G_b, \phi_b \) are phase angles of \( I^G_b, I_b \).

\[ P_{Loss}^L = \sum_{b=1}^m p_{lossb}^L \]

Therefore, the total transmission loss allocation to load L is computed as follows:

\[ P_{Loss}^L = \sum_{b=1}^m P_{lossb}^L \]

**ii) Transmission Loss Allocation to Generators**

Loss due to mismatch in generator’s voltage are obtained by setting the load currents to zero. Setting \( I_L \) in Eqn. (14) to zero, and keeping the generator voltage as obtained from the power flow solution. The current injection at generator bus is shown by IG, its contributions to the load bus voltage will be calculated according to below:

\[ V^L_k = (Y^{-1}_{LL} Y_{LG} Z_{GG}) e^{j \phi^L_G} \]

Where, \( Z_{GG} = ([Y_{GG}] - [Y_{GL}] [Y_{LL}] -1 [Y_{LG}] -1) \)

Also the contribution of IG to the generator bus voltage can be computed as:

\[ V^G_k = Z_{GG} e^{j \phi^G_G} \]

The contribution of IG to the current through branch can be computed as:

\[ I^G_b = \frac{\Delta V^G_k}{Z_b} \]

According to Superposition principle the branch current contributions due to each generator current injection is calculated as:

\[ I_b = \sum_{G=1}^G I^G_b \]

**B. Un-Subsidized LFD Method**

**i) Transmission Loss Allocation to Loads**

If losses allocated by subsidized method (PLs) to any load bus is less than „0” i.e., negative value, then,

\[ \beta = 1 / (1 - K_i) \]

where, \( K_i \) is the negative loss of ith branch allocated to the load buses and

\[ P_{Lu} = P_{Lu} \times \beta + (1 - \beta) \]

where, \( P_{Lu} \) is loss allocated by un-subsidized method

**ii) Transmission Loss Allocation to Generators**

If losses allocated by subsidized method (PGs) to any generator bus is less than „0” i.e., negative value, then,

\[ \mu = 1 / (1 - K_i) \]

where, \( K_i \) is the negative loss of ith branch allocated to the generator buses

\[ PTGs = \text{Total subsidized losses at particular branch} \]

**V. CASE STUDY–I: IEEE 5-BUS SYSTEM**

The IEEE 5 – Bus System shown in Fig. 1 is used to compare the transmission loss allocation methods considered in this paper. It consists of two generators (G1 and G2) and three loads (L3, L4 and L5) respectively. The 5 bus system represented by the
bus power injections, line power flows and line power losses obtained from the base case solution i.e., Newton Raphson load flow analysis.

A. Theoretical calculations for PS Method

The transmission losses can be allocated to the generators using Eqn. (1), i.e.,

Loss Allocation to Generators 1 = (1/2)* 3.36* (103.36/183.36) = 0.947 MW

Similarly, loss allocation to Generator 2 is 0.733 MW

The transmission losses can be allocated to the loads using Eqn. (2), i.e.,

Loss Allocation to Load 3 = (1/2)* 3.36* (60/180) = 0.947 MW

Loss Allocation to Generator 2 is 0.733 MW

Similarly, loss allocation to load 4 and 5 are 0.47 MW and 0.65 MW

B. Theoretical Calculations for PSP Method

The adjacency matrix (D) can be formed using Eqn. (3) as follows

\[ D = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \]

The branch flow matrix (Fd) can be calculated using Eqn. (4) as follows

\[ F_d = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \]

The nodal powers can be calculated using Eqn. (5) as follows

\[ P = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \]

The upstream and downstream allocation matrices obtained from the Eqns. (6) and (7) are

\[ A_M = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-0.1840 & -0.5178 & 1 & 0 & 0 & 0 & 0 \\
-0.8160 & -0.1217 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -0.3606 & 0 & -0.4605 & 1 & 0
\end{bmatrix} \]

\[ A_d = \begin{bmatrix}
1 & 0 & -0.3126 & -0.8957 & 0 & 0 & 0 \\
0 & 1 & -0.6874 & -0.1043 & -0.4016 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -0.5984 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \]

Using Eqn. (8), allocation of generators to all loads are computed as Power from Gen.1 to Load 3
\[ P_{g31} = \frac{60}{60} \times 0.1840 \times 103.3609 = 19.0175 \text{ MW} \]

Similarly

\[ P_{g32} = 41.4203 \text{ MW}, P_{g42} = 5.2515 \text{ MW}, P_{g52} = 33.3282 \text{ MW} \]

Using Eqn. (9), allocation of loads to all generators are computed as

\[ P_{D13} = \frac{103.3609}{103.3609} \times 0.3126 \times 60 = 18.7569 \text{ MW} \]

Similarly

\[ PD13 = \frac{103.3609}{103.3609} \times 0.3126 \times 60 = 18.7569 \text{ MW} \]

Now, the gross power injected to the generators and loads are computed as

\[ P_{G1} = PD13 = 18.7569 + 44.7863 + 37.5204 = 101.0636 \text{ MW} \]

\[ P_{G2} = PD23 = 41.2431 + 5.2137 + 32.4796 = 78.9364 \text{ MW} \]

\[ P_{G3} = PD3 = 45.5027 + 5.2515 = 50.7542 \text{ MW} \]

\[ P_{G4} = PD4 = 41.2431 + 5.2137 + 32.4796 = 78.9364 \text{ MW} \]

Using Eqns. (10) and (11) the final generation and load per bus are computed as

\[ P'_{G1} = \frac{P_{G1} + P_{G1}}{2} = \frac{101.0636 + 103.3609}{2} = 102.2122 \text{ MW} \]

Similarly \( P'_{D2} = 79.4682 \text{ MW} \)

\[ P'_{D3} = \frac{P_{D3} + P_{D3}}{2} = \frac{60.4378 + 60}{2} = 60.2189 \text{ MW} \]

Similarly,

\[ P'_{D4} = 50.3771 \text{ MW}, P'_{D5} = 71.0844 \text{ MW} \]

Using Eqns. (12) and (13) transmission loss allocation to generators and demands is computed as

\[ L'_{G1} = P_{G1} - P'_{G1} = 103.3609 - 102.2122 = 1.1487 \text{ MW} \]

\[ L'_{G2} = P_{G2} - P'_{G2} = 80 - 79.4682 = 0.5318 \text{ MW} \]

\[ L'_{D3} = P_{D3} - P'_{D3} = 60.2189 - 60 = 0.2189 \text{ MW} \]

Similarly,

\[ L'_{D4} = 0.3771 \text{ MV} \]

**C. Theoretical Calculations for LFD Method**

Loss Allocation to Loads: Y-Bus is formed for 5-bus system shown in Fig. 1 as follows

\[
Y = \begin{bmatrix}
60.25 & -60 & 0 & -125 + j37.75 & 0 \\
-60 & 120.47 & -60 & 112.5 - j37.75 & 0 \\
0 & 120.47 & 120.47 & -60 & -125 + j37.75 \\
-125 + j37.75 & 112.5 - j37.75 & -60 & 120.47 & 0 \\
0 & 125 - j37.75 & -125 + j37.75 & 120.47 & 120.47
\end{bmatrix}
\]

The current injection values to the loads can be calculated by using Eqn. (14) and keeping \( V_G = 0 \)

\[
\begin{bmatrix}
I_{L3} \\
I_{L4} \\
I_{L5}
\end{bmatrix} =
\begin{bmatrix}
11.25 - j33.71 & 0 & 0 \\
0 & 9.16 - j27.43 & -25 + j7.5 \\
0 & -25 + j7.5 & 3.75 - j11.21
\end{bmatrix}
\begin{bmatrix}
-0.01 - j0.013 & 0 & 0 \\
0 & -0.011 - j0.17 & 0 \\
0 & 0 & -0.057 - j0.05
\end{bmatrix}
\]

Now the current injection values are

\[ I_{L3} = -0.56 + j0.214, I_{L4} = -0.47 + j0.117, I_{L5} = -0.672 + j0.359 \]

The contribution of load bus currents to the load bus voltages can be calculated using Eqn. (15)

Now the contribution of IL3 to the load bus voltages is as follows

\[
\begin{bmatrix}
V_{L3}^4 \\
V_{L3}^5
\end{bmatrix} =
\begin{bmatrix}
0.0089 + j0.026 & 0 & 0 \\
0 & 0.013 + j0.04 & 0.009 + j0.027 \\
0 & 0.09 + j0.026 & 0.033 + j0.098
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\begin{bmatrix}
-0.567 + j0.214
\end{bmatrix}
\]

Now
\[V_{3}^{L3} = -0.0108 - j0.0132, V_{4}^{L3} = 0 + j0, V_{5}^{L3} = 0 + j0\]

The contribution of IL3 to the voltage drop across branch is computed using Eqn. (16) as
\[\Delta V_{13} = V_{1} - V_{3} = 0.0108 + j0.0132,\]
\[\Delta V_{14} = V_{1} - V_{4} = 0 + j0,\]
\[\Delta V_{23} = V_{2} - V_{3} = 0.0108 + j0.0132,\]
\[\Delta V_{24} = V_{2} - V_{4} = 0 + j0,\]
\[\Delta V_{25} = V_{2} - V_{5} = 0 + j0\] and \[\Delta V_{45} = V_{4} - V_{5} = 0 + j0\]

The contribution of IL3 to the current through branch can be computed using Eqn. (17) as:

\[I_{1}^{L3} = \frac{\Delta V_{13}}{Z_{13}} = 0.06314 - j0.0238\]
\[I_{2}^{L3} = \frac{\Delta V_{14}}{Z_{14}} = 0 + j0\]
\[I_{3}^{L3} = \frac{\Delta V_{23}}{Z_{23}} = 0.5051 - j0.191\]
\[I_{4}^{L3} = \frac{\Delta V_{24}}{Z_{24}} = 0.5051 - j0.191\]
\[I_{5}^{L3} = \frac{\Delta V_{25}}{Z_{25}} = 0 + j0\] and \[I_{6}^{L3} = \frac{\Delta V_{45}}{Z_{45}} = 0 + j0\]

Similarly the contribution of IL4 to the load bus voltages is as follows
\[V_{3}^{L4} = 0 + j0, V_{4}^{L4} = 0.011 - j0.018, V_{5}^{L4} = -0.0076\]

The contribution of IL4 to the voltage drop across branch is
\[\Delta V_{13} = V_{1} - V_{3} = 0 + j0, \Delta V_{14} = V_{1} - V_{4} = 0.011 + j0.018\]
\[\Delta V_{23} = V_{2} - V_{3} = 0 + j0, \Delta V_{24} = V_{2} - V_{4} = 0.011 + j0.018\]
\[\Delta V_{25} = V_{2} - V_{5} = 0.057 + j0.054\] and \[\Delta V_{45} = V_{4} - V_{5} = 0.041 + j0.054\]

The contribution of IL4 to the current through branch is as follows
\[I_{1}^{L4} = \frac{\Delta V_{12}}{Z_{12}} = 0 + j0, I_{2}^{L4} = \frac{\Delta V_{13}}{Z_{13}} = 0.319 - j0.078\]
\[I_{3}^{L4} = \frac{\Delta V_{24}}{Z_{24}} = 0.106 - j0.026, I_{4}^{L4} = \frac{\Delta V_{25}}{Z_{25}} = 0.053 - j0.013\]
\[I_{5}^{L4} = \frac{\Delta V_{34}}{Z_{34}} = 0.5051 - j0.191\] and \[I_{6}^{L4} = \frac{\Delta V_{45}}{Z_{45}} = 0.347 - j0.214\]

The branch current contribution due to each current injection can be evaluated using Eqn. (18) is as follow
\[I_{1}^{L3} + I_{2}^{L3} + I_{3}^{L3} + I_{4}^{L3} = 0.063 - j0.023, I_{2}^{L4} = 0.619 - j0.240\]
\[I_{3}^{L3} + I_{4}^{L3} + I_{5}^{L3} = 0.505 - j0.191, I_{4}^{L4} + I_{5}^{L4} = 0.207 - j0.080\]
\[I_{5}^{L3} + I_{6}^{L3} + I_{7}^{L3} = 0.328 - j0.160\] and \[I_{6}^{L4} + I_{7}^{L4} = 0.347 - j0.201\]

Now, the loss (in MW) allocation to all the branches can be calculated using Eqn. (21) For current injection at load 3 PlossL3 = 0.094, Ploss2L3 = 0, Ploss3L3 = 0.227, Ploss4L3 = 0, Ploss5L3 = 0, Ploss6L3 = 0.

For current injection at load 4:
\[PlossL4 = 0.537, Ploss3L4 = 0, Ploss4L4 = 0.130, Ploss6L4 = -0.095.\]

For current injection at load 5:
Ploss1 L5 = 0, Ploss2L5 = 0.534, Ploss3L5 = 0,
Ploss4L5 = 0.073, Ploss5L5 = 0.755, Ploss6L5 = 0.804.

The total transmission loss allocation to each load is computed using Eqn. (22) as follows

Loss allocation to load 3:
\[ PT_{loss,L3} = Ploss1L3 + Ploss2L3 + Ploss3L3 + Ploss4L3 + Ploss5L3 + Ploss6L3 \]
\[ = 0.094 + 0 + 0.227 + 0 + 0 + 0 = 0.321 \text{ MW} \]

Loss allocation to load 4:
\[ PT_{loss,L4} = Ploss1L4 + Ploss2L4 + Ploss3L4 + Ploss4L4 + Ploss5L4 + Ploss6L4 \]
\[ = 0 + 0.537 + 0 + 0.067 + 0.130 - 0.095 = 0.639 \text{ MW} \]

Loss allocation to load 5:
\[ PT_{loss,L5} = Ploss1L5 + Ploss2L5 + Ploss3L5 + Ploss4L5 + Ploss5L5 + Ploss6L5 \]
\[ = 0 + 0.534 + 0 + 0.227 + 0.755 + 0.804 \]
\[ = 2.166 \text{ MW} \]

Loss Allocation to Generators: The current injections at generator buses are as follows:
\[ IG1 = 0.291+j0.127 \text{ and } IG2 = -0.287+j0.137 \]

The contribution of IG1 to the load bus voltages using Eqn. (23) are:
\[ V3G1 = 0.508 - j1.170, \]
\[ V4G1 = 0.508 - j1.164, \]
\[ V5G1 = 0.5087 - j1.1709 \]

The voltage contributions of generator due to generator current injections are computed using Eqn. (24) as follows:
\[
\begin{bmatrix}
V_{G1}^G \\
V_{G2}^G
\end{bmatrix}
= \begin{bmatrix}
0.012 + j3.962 & -0.006 - j0.039 \\
-0.006 - j4.019 & 0.011 - j3.967
\end{bmatrix}
\begin{bmatrix}1 \\ 0.291 + j0.127\end{bmatrix}
\]

\[ V_{G1}^G = 0.507 - j1.152 \text{ and } V_{G2}^G = 0.508 - j1.171 \]

Now the contribution of IG1 to the voltage drop across branch is computed as
\[ \Delta V_{13} = V1 - V3 = -0.0016 + j0.017, \]
\[ \Delta V_{14} = V1 - V4 = -0.001 + j0.011 \]
\[ \Delta V_{23} = V2 - V3 = 0.00008 - j0.00091, \]
\[ \Delta V_{24} = V2 - V4 = 0.0006 - j0.0077 \]
\[ \Delta V_{25} = V2 - V5 = 0.00006 - j0.000 \text{ and } \]
\[ \Delta V_{45} = V4 - V5 = -0.0006 + j0.0068 \]

The contributions of IG1 to the current through each branch are computed using Eqn. (25) as
\[ I_{1G1} = \Delta V_{13}/Z_{13} = 0.065 + j0.028, \]
\[ I_{2G1} = \Delta V_{14}/Z_{14} = 0.162 + j0.070 \]
\[ I_{3G1} = \Delta V_{23}/Z_{23} = -0.026 + j0.011, \]
\[ I_{4G1} = \Delta V_{24}/Z_{24} = -0.037 - j0.016 \]
\[ I_{5G1} = \Delta V_{25}/Z_{25} = -0.0029 - j0.0012 \]
\[ I_{6G1} = \Delta V_{45}/Z_{45} = 0.049 + j0.022 \]

Similarly, the contribution of IG2 to the load bus voltages and generator bus voltages are
\[ V1G2 = 0.552 + j1.152, V2G2 = 0.540 + j1.140 \]
\[ V3G2 = 0.542 - j1.142, V4G2 = 0.552 - j1.152 \]
\[ V5G2 = 0.551 - j1.151 \]

Now the contribution of IG2 to the voltage drop across branch is given as:
\[ \Delta V_{13} = V1 - V3 = 0.011 + j0.0102, \]
\[ \Delta V_{14} = V1 - V4 = 0.0006 + j0.00063 \]
\[ \Delta V_{23} = V2 - V3 = 1.138 - j1.142, \]
\[ \Delta V_{24} = V2 - V4 = 1.128 - j1.152 \]
\[ \Delta V_{25} = V2 - V5 = 1.129 - j1.151 \]
\[ \Delta V_{45} = V4 - V5 = 0.0009 + j0.0008 \]

The contribution of IG2 to the current through each branch is given as:
I1 = ΔV13/Z13 = 0.508-j0.024,
I2 = ΔV14/Z14 = 0.013-j0.006
I3 = ΔV23/Z23 = 0.090+j0.044,
I4 = ΔV24/Z24 = 0.078+j0.037
I5 = ΔV25/Z25 = 0.055+j0.026 &
I6 = ΔV45/Z45 = 0.0086-j0.0042

The branch current contribution due to each current injection can be evaluated using Eqn. (26) as follow:

\[ I_{1} = I_{1L1} + I_{1L2} + I_{1L3} = 0.574+j0.004, \]
\[ I_{2} = I_{2L1} + I_{2L2} + I_{2L3} = 0.018+j0.064 \]
\[ I_{3} = I_{3L1} + I_{3L2} + I_{3L3} = 0.117+0.032, \]
\[ I_{4} = I_{4L1} + I_{4L2} + I_{4L3} = 0.116+j0.021 \]
\[ I_{5} = I_{5L1} + I_{5L2} + I_{5L3} = 0.058+j0.025 \]
\[ I_{6} = I_{6L1} + I_{6L2} + I_{6L3} = 0.058+j0.017 \]

Now, the loss (in MW) allocation to all the branches can be calculated using Eqn. (28).

For current injection at Generator 1:
\[ P_{loss1G1} = 0.089, \quad P_{loss2G1} = 0.233, \quad P_{loss3G1} = -0.007, \]
\[ P_{loss4G1} = -0.014, \quad P_{loss5G1} = -0.005, \quad P_{loss6G1} = 0.064. \]

For current injection at Generator 2:
\[ P_{loss1G2} = 0.076, \quad P_{loss2G2} = 0.022, \quad P_{loss3G2} = -0.042, \]
\[ P_{loss4G2} = -0.056, \quad P_{loss5G2} = -0.147, \quad P_{loss6G2} = 0.017. \]

The total transmission loss allocation to each generator is computed using Eqn. (29) as follows:

Loss allocation to Generator 1:
\[ P_{TlossG1} = P_{loss1G1} + P_{loss2G1} + P_{loss3G1} + P_{loss4G1} + P_{loss5G1} + P_{loss6G1} = 0.089 + 0.233 - 0.007 - 0.014 - 0.005 + 0.064 = 0.36 \text{ MW}. \]

Loss allocation to Generator 2:
\[ P_{TlossG2} = P_{loss1G2} + P_{loss2G2} + P_{loss3G2} + P_{loss4G2} + P_{loss5G2} + P_{loss6G2} = 0.076 + 0.022 - 0.000 + 0.000 - 0.000 + 0.017 = 0.12 \text{ MW}. \]

D. Transmission Loss Allocation Results

Table I shows the transmission power losses allocated to the generators and the Table II shows the transmission power losses allocated to the loads using three loss allocation methods.

**TABLE I**

<table>
<thead>
<tr>
<th>Generator no</th>
<th>PS Method</th>
<th>PSP Method</th>
<th>LFD Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9972</td>
<td>1.2219</td>
<td>0.3600</td>
</tr>
<tr>
<td>2</td>
<td>0.7705</td>
<td>0.5458</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

**TABLE II**

<table>
<thead>
<tr>
<th>Load no</th>
<th>PS Method</th>
<th>PSP Method</th>
<th>LFD Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5892</td>
<td>0.2554</td>
<td>0.38587</td>
</tr>
<tr>
<td>2</td>
<td>0.4910</td>
<td>0.4072</td>
<td>0.71711</td>
</tr>
<tr>
<td>3</td>
<td>0.6874</td>
<td>1.1050</td>
<td>2.2442</td>
</tr>
</tbody>
</table>

From the above results, it can be observed that PS method could not consider the network into account, it allocate the losses to the generators and demands with respect to their capacities with out considering the power flows between them, from the results it is can be seen that the PS method allocates the losses of 0.95MW to the Generator -1, but its contribution of power flow to the other demands apart from its own demand is only 103.36MW and it allocates the losses of 0.73MW to the generator – 2 with its contribution of 80MW power flow to the other demands.
The PSP method is considering the network into account; it allocates the losses to the generators and demands according to their power flow contribution. From the results it shows that the PSP method allocates the losses of 1.15MW to the generator-1 as it contributes the 103.36MW to the other demands and it allocates the 0.53MW losses to the generator – 2 as it contributes the 80MW power flow to the other demands.

From the results, it is observed that the LFD method allocates the losses with respect to losses occurred in each segment. This method will not consider the assumption of allocating 50% losses to the generators and 50% to the loads. The loss allocated to all generators is 0.23MW and loads are 3.13MW. So, from this analysis it is observed that, with the LFD method the entire transmission network customers are reasonably benefited.

VI. CASE STUDY–II: IEEE 14-BUS SYSTEM

A case study based on the IEEE, shown in Fig. 2, is presented in this section.

Fig. 2 IEEE 14 – Bus System

A. Transmission Loss Allocation Results

The results of IEEE – 14 bus system obtained for the three loss allocation methods are shown in the tables. Table III shows the transmission power losses allocated to the generators and the Table IV shows the transmission power losses allocated to the loads using three cost allocation methods.

<table>
<thead>
<tr>
<th>Generator no</th>
<th>PS Method</th>
<th>PSP Method</th>
<th>LFD Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.8726</td>
<td>6.2308</td>
<td>7.2505</td>
</tr>
<tr>
<td>2</td>
<td>1.0092</td>
<td>0.60596</td>
<td>0.03448</td>
</tr>
</tbody>
</table>

VI. CONCLUSIONS

Application of the proposed method is straightforward. It requires only a solved power flow and any simple algorithm for power flow tracing. Both active and reactive powers are considered in the loss allocation procedure. Results of application show the accuracy of the proposed method compared with the commonly used procedures. The Proportional Sharing Principle method provides fair and efficient loss allocation to the users when compare to Postage Stamp Method which allocates fixed costs to the users. But, this PSP method follows the assumption of allocating 50% losses to the generators and 50% to the loads.

The LFD method will not consider this assumption. The Postage Stamp method does not take into account the network and produces substantially different results than other methods. The proportional sharing procedure takes into account the network. The Loss Function Decomposition method provides fair, efficient and accurate loss allocation to the transmission system customers when compare to the Postage Stamp method which allocates fixed and equal loss to both generators and loads. This method will not consider the assumption of allocating 50% losses to the generators and 50% to the loads. Postage Stamp method is not advisable because it is unfair for specific group of generators and demands. Generators close to load centre are unfairly treated with respect to generators far away from load center. Similarly, demands close to generating areas are unfairly treated with respect to demands far away from those areas.
REFERENCES


