A NEW APPROACH FOR IMAGE RESTORATION BASED ON JSM IN A SPACE TRANSFORM DOMAIN

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ABSTRACT

In general a completely unique strategy for hi-fi image restoration by characterizing each local smoothness and nonlocal self-similarity of natural pictures during a unified applied mathematics manner. the most contributions area unit three-fold. First, from the angle of image statistics, a joint applied mathematics modeling (JSM) in AN adjective hybrid space-transform domain is established, that offers a {robust} mechanism of mixing native smoothness and nonlocal self-similarity at the same time to confirm a a lot of reliable and robust estimation. Second, a brand new sort of step-down practical for finding the image inverse drawback is developed victimization JSM beneath a regularization-based framework. Finally, so as to create JSM tractable and strong, a brand new Split Bregman-based rule is developed to expeditiously solve the on top of severely underdetermined inverse drawback related to theoretical proof of convergence. intensive experiments on image inpainting, image deblurring, and mixed Gaussian and salt-and pepper noise removal applications verify the effectiveness of the projected rule.

INTRODUCTION

The presence of noise in images is inescapable. it's going to be introduced by the image formation method, image recording, image transmission, etc. These random distortions create it make to perform any needed image process. as an example, the feature directed ,Sweetening is incredibly effective in restoring hazy pictures however it may be "frozen" by associate oscillating noise part. Even a little quantity of noise is harmful once high accuracy is needed, e.g. as in subcell (subpixel) image analysis.

In apply, to estimate a real signal in noise, the most oftentimes used ways area unit supported the least squares criteria. The principle comes from the statistical argument that the least squares estimation is that the best over a whole ensemble of all possible pictures.

As a basic drawback within the field of image processing, image restoration has been
extensively studied within the past twenty years. It aims to reconstruct the original high-quality image \( x \) from its degraded discovered version \( y \), that could be a typical ill-posed linear inverse drawback and can be usually developed as

\[
y = Hx + n \quad \ldots \ldots (1)
\]

where \( x, y \) are lexicographically stacked representations of the original image and the degraded image, respectively; \( H \) is a matrix representing a noninvertible linear degradation operator; and \( n \) is usually additive Gaussian white noise. When \( H \) is identity, the problem becomes image de-noising once \( H \) is a blur operator, the problem becomes image deblurring when \( H \) could be a mask, that is, \( H \) could be a diagonal matrix whose diagonal entries are either one or zero, keeping or killing the corresponding pixels, the problem becomes image inpainting when \( H \) is a group of random projections, the problem becomes compressive sensing during this paper, we tend to specialize in image inpainting, image deblurring, and image denoising.

In order to deal with the ill-posed nature of image restoration, one kind of theme in literature employs previous data of a figure for regularizing the solution to the subsequent diminution problem ing.

\[
\text{argmin}_x \frac{1}{2} \|Hx - y\|_2^2 + \lambda \psi(x) \quad \ldots \ldots (2)
\]

Where \( \frac{1}{2} \|Hx - y\|_2^2 \) is the \( \ell_2 \) data-fidelity term \( \psi(x) \) is called the regularization term denoting image prior, and \( \lambda \) is the regularization parameter. In fact, the above regularization-based framework (2) can be strictly derived from Bayesian inference with some image prior possibility model. Many optimization approaches for regularization-based image inverse problems have been developed. It has been widely recognized that image prior knowledge plays a critical role in the performance of image-restoration algorithms.

Based on the studies of previous work, 2 shortcomings are discovered. On one hand, just one image property utilized in regularization-based framework isn’t enough to get satisfying restoration results. On the opposite hand, the image property of nonlocal self-similarity ought to be characterized by a additional powerful manner, instead of by the normal weighted graph. During this paper, we tend to propose a completely unique strategy for accurate image restoration by characterizing each native smoothness and nonlocal self-similarity of natural pictures in a unified applied math manner. Our main contributions area unit listed as follows. First, from the angle of image statistics, we tend to establish a joint statistical modeling (JSM) in AN adaptive hybrid house and remodel domain, that offers sturdy} mechanism of mixing native smoothness and nonlocal self similarity at the same time to make sure a additional reliable and robust estimation.

Second, a new sort of reduction functional for resolution image inverse issues is developed
victimization JSM underneath regularization-based framework. The planned methodology may be a general model that has several connected models as special cases. Third, so as to form JSM tractable and strong, a new Split Bregman-based rule is developed to expeditiously solve the on top of severely underdetermined inverse downside related to theoretical proof of convergence.

PROPOSED METHOD

To deal with the ill-posed nature of image inverse problems, previous knowledge regarding natural pictures is usually utilized, specifically image properties, that basically play a key role in achieving high-quality pictures. Here, 2 kinds of fashionable image properties are thought of, specifically native smoothness and nonlocal self-similarity, as illustrated by image Lenain Fig. 1. the previous sort describes the piecewise smoothness among native region, as shown by circular regions, whereas the latter one depicts the verboseness of the textures or structures in globally positioned image patches, as shown by block regions with identical color.

JSM is established by merging two complementary models:

- Local statistical modeling (LSM) in 2D space domain and
- Nonlocal statistical modeling (NLSM) in 3D transform domain, that is

$$\psi_{JSM}(u) = \tau \cdot \psi_{LSM}(u) + \lambda \cdot \psi_{NLSM}(u)$$

Where $\tau$, $\lambda$ are regularization parameters, which control the tradeoff between two competing statistical terms. $\psi_{LSM}$ corresponds to the above local smoothness prior and keeps image local consistency, suppressing noise effectively, while $\psi_{NLSM}$ corresponds to the above nonlocal self-similarity prior and maintains image nonlocal consistency, retaining the sharpness and edges effectively. More details on how to design JSM to characterize the above two properties will be provided below.

A. Local Statistical Modeling for Smoothness in Space Domain

Local smoothness describes the closeness of neighboring pixels in 2D space domain of images, which means the intensities of the neighboring pixels are quite similar. To characterize the smoothness of images, there exist many models. Here, we mathematically formulate a local statistical modeling for smoothness in 2D space domain. From the view of statistics, a natural image is preferred when its responses for a set of high passing filters are as small as possible which intuitively implies that
images are locally smooth and their derivatives are close to zero

\[ p_{GGD}(x) = \frac{\nu \eta(v)}{2\Gamma(1/v)} \frac{1}{\sigma_x} e^{-\left[\frac{\eta(v) |x|}{\sigma_x}\right]^v} \]

\[ \eta(v) = \sqrt{\Gamma(3/v)\Gamma(1/v)} \]

and \( \Gamma(t) = \int_0^\infty e^{-u} u^{t-1} du \)

\[ \text{Where } v \text{ is the shape parameter. The distribution } p_{GGD}(x) \text{ is a Gaussian distribution function if } v = 2 \text{ and a Laplacian distribution function if } v=1. \]

If \( 1 < v < 2 \), \( p_{GGD}(x) \) is named as a hyperLaplacian distribution we choose Laplacian distribution to model the marginal distributions of gradients of natural images by making a tradeoff between modeling the image statistics accurately and being able to solve the ensuing optimization problem efficiently. Thus, let \( D = [D_v; D_h] \) and set \( v = 1 \) in (4) to obtain LSM in space domain at pixel level, with corresponding regularization term

\[ \Psi_{LSM} \]

\[ \Psi_{LSM}(u) = \|D_u\|_1 = \|D_v(u)\|_1 + \|D_h(u)\|_1 \]

\[ \text{figure} \]

which clearly indicates that the formulation is convex and facilitates the theoretical analysis.

Note that LSM has the same expression as anisotropic TV can be regarded as a statistical interpretation of anisotropic TV. It is important to emphasize that local statistical modeling is only used for characterizing the property of image smoothness. The regularization term (5) has the advantages of convex optimization and low computational complexity. There is no need to design a very complex regularization term, since the task of retaining the sharp edges and recovering the fine textures will be accomplished by the following nonlocal statistical modeling. More details for solving LSM regularized problems will be given in the next section.

**B. Nonlocal Statistical Modeling for Self-Similarity in Transform Domain**
Besides local smoothness, nonlocal self-similarity is another significant property of natural images. It characterizes the repetitiveness of the textures or structures embodied by natural images within nonlocal area, which can be used for retaining the sharpness and edges effectively to maintain image nonlocal consistency. The mathematical formulation of nonlocal statistical modeling for self-similarity in 3D transform domain is written as

$$\psi_{\text{NLSM}}(u) = \|\Theta u\|_1 = \sum_{i=1}^n \|T^{3D}(Z_{u^i})\|_1$$

......................(6)

### C. Joint Statistical Modeling (JSM)

Considering local smoothness and nonlocal self-similarity in a whole, a new JSM can be defined by combining the LSM for smoothness in space domain at pixel level and the NLSM in transform domain at block level, which is expressed as

$$\psi_{\text{JSM}}(u) = \tau \cdot \psi_{\text{LSM}}(u) + \lambda \cdot \psi_{\text{NLSM}}(u) = \tau \cdot \|\Delta u\|_1 + \lambda \cdot \|\Theta u\|_1$$

......................(7)

Thus, JSM is able to portray local smoothness and nonlocal self-similarity of natural images richly, and combine the best of the both worlds, which greatly confines the space of inverse problem solution and significantly improve the reconstruction quality. To make JSM tractable and robust, a new Split Bregman-based iterative algorithm is developed to solve the optimization problem with JSM as regularization term efficiently, whose implementation details and convergence proof will be provided in the next section. Extensive experimental results will testify the validity of the proposed JSM.

### Algorithm 1 Split Bregman Iteration (SBI)

1. Set $k = 0$, choose $\mu > 0$,
   $$d^{(0)} = 0, u^{(0)} = 0, v^{(0)} = 0.$$  
2. Repeat
   3. $u^{(k+1)} = \arg\min_u f(u) + \frac{\mu}{2} \left\| Gu - y - d^{(k)} \right\|_2^2$;
   4. $v^{(k+1)} = \arg\min_v g(v) + \frac{\mu}{2} \left\| Gu^{(k+1)} - v - d^{(k)} \right\|_2^2$;
   5. $d^{(k+1)} = d^{(k)} - (Gu^{(k+1)} - v^{(k+1)})$;
   6. $k \leftarrow k + 1$;
   7. Until stopping criterion is satisfied

### Split Bregman-Based Iterative Algorithm for Image Restoration Using JSM

By incorporating the proposed joint statistical modeling (7) into the regularization-based framework (2), a new formulation for image restoration can be expressed as

$$\arg\min_u \frac{1}{2} \|Hu - y\|_2^2 + \tau \cdot \psi_{\text{JSM}}(u) + \lambda \cdot \psi_{\text{NLSM}}(u)$$

......................(8)

where $\tau$ and $\lambda$ are management parameters. Note that the primary term of (8) truly represents the observation constraint and therefore the second and therefore the third represent the image previous native and nonlocal constraints, severally

**Theorem 1:** The proposed algorithm described by Table I converges to a solution of (8).

**Proof:** It is obvious that the proposed algorithm is an instance of SBI. Since all the three
functions are closed, proper, and convex, the convergence of the proposed algorithm is guaranteed
\[ G = \begin{bmatrix} I \\ I \end{bmatrix} \in \mathbb{R}^{2N \times N} \]

**Theorem 2:**

Let \( x, r \in \mathbb{R}^N \), \( x, r \in \mathbb{R}^K \), and denote the error vector by \( e = x - r \) and each element of \( e \) by \( e(j) \), \( j = 1, ..., N \). Assume that \( e(j) \) is independent and comes from a distribution with zero mean and variance \( \sigma^2 \). Then, for any \( \varepsilon > 0 \), we have the following property to describe the relationship
\[
\lim_{N \to \infty} P \left\{ \frac{1}{N} \sum_{j=1}^{N} e(j)^2 < \varepsilon^2 \right\} = 1,
\]

orthogonal property of transform T 3D as follows
\[
\lim_{N \to \infty} P \left\{ \frac{1}{N} \left\| x - r \right\|^2_2 - \sigma^2 < \varepsilon^2 \right\} = 1
\]

Incorporating (26) into (21) leads to
\[
x = \frac{1}{2} \left\| \Theta_x - \Theta_r \right\|^2_2 + \frac{k\alpha}{N} \left\| \Theta_x \right\|_1
\]

Since the unknown variable \( x \) is component-wise separable in (27), each of its components \( x(j) \) can be independently obtained in a closed form according to the so-called soft thresholding
\[
\theta_x = \text{soft} (\theta_r, \sqrt{2\rho})
\]
Simulation results:

![Input deblurred image](image1)

![Input deblurred image](image2)

![Output image](image3)
CONCLUSION

In this paper, a unique algorithmic program for high-quality image restoration exploitation the joint statistical modeling in a very space transform domain is projected, that with efficiency characterizes the intrinsic properties of native smoothness and nonlocal self similarity of natural pictures from the perspective of statistics at an equivalent time. Experimental results on 3 applications: image inpainting, image deblurring, and mixed Gaussian and salt-and-pepper noise removal have shown that the planned algorithm achieves vital performance enhancements over the current state-of-the-art schemes and exhibits nice convergence property. Future work includes the investigation of the statistics for natural pictures at multiple scales and orientations and the extensions on a range of applications, like image deblurring with mixed Gaussian and impulse noise and video restoration tasks deblurring with mixed Gaussian and impulse noise and video restoration tasks.

REFERENCES


