A Novel Weighted Subspace Fitting CFO Estimation In The OFDM Systems

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ABSTRACT
Whenever we are designing the wireless communication systems that must possess of high reliability and low latency and also the optimization when switching is takes place in different circuits. and in that system it contains of lot of hardware impairments regarding power supply . when the supply load changes in circuit suddenly the carrier frequency drifts it results in transient carrier frequency offset(CFO) in cannot be estimated by using of the traditional estimators and is typically addressed by inserting or extending guard intervals. In this paper we are presenting the modeling and estimation of the transient CFO , which is modeled as the response of an under damped second order system. To compensate for the transient CFO, we are proposed a low complexity parametric estimation algorithm by using of the null space of Hankel-like matrix. It was constructed from the phase difference of the two halves of the repetitive training sequence.

Here a weighted subspace fitting algorithm is derived with a slight increase in complexity to minimize the mean squared error of the estimated parameters in noise.

1.INTRODUCTION
Since cooperation was first projected for wireless networks, it's become a preferred area of communications and networking analysis. The performance gains for cooperation stem from mitigating long path-loss and shadowing effects and short multipath fading effects. The combined path-loss of a two-hop transmission is also but direct transmission, and RF absorbing objects within the atmosphere are often circumvented to scale back shadowing. this can be analogous to multi-hop routing and might be performed at either network (NET) or medium access (MAC) layer, with larger potency at the latter. Relaying also can give a receiver with redundant messages that travel through spatially distinct ways, reducing the impact of multipath. This creation of special diversity, referred to as cooperative diversity, provides better average performance and might be achieved at either the
mack or physical (PHY) layer, with larger performance gains at the latter. The tip results of path-loss and variety gains provided through cooperation is improved network performance, realizable as varied combinations of power savings, rate will increase, network coverage growth, and interference reduction. A number of characteristics of cooperative schemes make experimentation essential for transitioning proposed schemes to practical applications. Information flowing through a cooperative link visits multiple nodes.

Within the network, suggesting cooperation can have implications for multiple layers of the protocol stack, together with at a minimum the PHY, MAC, and internet layers. Decisions made at one layer can impact the potency of the others, increasing protocol quality however providing several opportunities for cross-layer improvement. Performance metrics to guage such cross-layer schemes, like output and delay at the mack and internet layers, need giant networks to get realistic estimates. Modeling and simulation will become unwieldy because the variety of nodes at intervals a network grows, creating experimentation “one of the factual approaches for benchmarking”. complicated interactions between layers will be not possible to fully predict, creating full implementation necessary to spot conflicts that will degrade performance.

Cooperative gains determined by theoretical work and simulation vary considerably relying upon the models chosen for the network topology and therefore the wireless channel. Real-world propagation environments area unit notoriously difficult to model; the foremost faithful models quickly become intractable for each theoretical work and simulation, increase this the multi-layer simulation needed by the cross-layer nature of cooperation, and experimentation becomes the sole sensible thanks to measure full protocols. Experimentation is a chance to get realistic quantifications of performance gains for planned cooperative schemes in actual propagation environments and network topologies.

Finally, modeling and simulation typically create idealistic assumptions regarding the capabilities of the physical radio platforms on that planned protocols ar to be run. Common limitations in real-world radios like a scarcity of frequency and temporal arrangement synchronization, imperfect channel estimation, and quantization errors have an oversized impact on the effectiveness of the many cooperative schemes. though a considerable quantity of the literature addresses these problems directly, experimentation is that the solely thanks to establish alternative impediments and make sure devised solutions are so solutions.

2. Proposed method:

2.1 The Traditional CFO Estimation:

We assume that the receiver estimates the CFO through the preamble OFDM symbol,
whose first and second halves are identical. Then, we can express the transmitted preamble signal as

\[
s(t) = \begin{cases} s_1(t) & 0 \leq t \leq T \\ s_1(t-T) & T \leq t \leq 2T \end{cases}
\]

(1)

Where \( T \) is the time interval between the two halves. Thus, the received signal impaired by CFO can be written as

\[
r(t) = \left( \int_{-\infty}^{+\infty} h(v) s(t-v) dv + n(t) \right) e^{j\Delta\omega t} + \tilde{n}(t)
\]

(2)

Where \( h(t) \) is the channel response, \( s(t) \) is the transmitted preamble and \( n(t) \) and \( \tilde{n}(t) \) are the additive Gaussian noise at the receiver before and after RF down-conversion, respectively. \( \Delta\omega(t) \) is the CFO between the transmitter and receiver. \( \tilde{n}(t) \) is much less than \( n(t) \). Hence, we drop this noise term thereafter. For the traditional CFO estimation, we assume that the CFO is constant, then (2) reduces to

\[
r(t) = \left( \int_{-\infty}^{+\infty} h(v) s(t-v) dv + n(t) \right) e^{j\Delta\omega t} 0 \leq t \leq T
\]

(3)

The first and second halves of the received preamble can be written as

\[
r_1(t) = \left( \int_{-\infty}^{+\infty} h(v) s(t-v) dv + n_1(t) \right) e^{j\Delta\omega t} 0 \leq t \leq T
\]

\[
r_2(t) = \left( \int_{-\infty}^{+\infty} h(v) s(t-v) dv + n_2(t) \right) e^{j\Delta\omega (t+T)} 0 \leq t \leq T
\]

Where \( n_1(t) \) and \( n_2(t) \) are the additive noise in \( r_1(t) \) and \( r_2(t) \), respectively. Therefore, the first and second halves of the received preamble symbol have a fixed phase difference of \( \Delta\omega \) in sample-wise with the absence of the additive noise. The traditional CFO estimation is to compute this fixed phase difference. The maximum likelihood estimation of the CFO, denoted by \( \hat{\Delta}\omega \), is

\[
\hat{\Delta}\omega = \cos \left( \sum_{n=0}^{N-2} r_1^*(nT_s) r_2(nT_s) \right) / T
\]

(4)

2.2 Transient CFO Model: Then, we claim that the transient CFO, denoted by \( \Delta\omega_T(t) \), can be model as the step response of a second order underdamped system, which can be expressed as an exponentially damped sinusoid, i.e.

\[
\Delta\omega_T(t) = \alpha e^{-\xi \omega_n t} \sin \left( \omega_n \sqrt{1 - \xi^2 t + \phi} \right) t > 0, 0 < \xi < 1
\]

(5)

Where \( \xi \) and \( \omega_n \) are the damping factor and undamped natural frequency, respectively. \( 0 < \xi < 1 \) means that the second order system is underdamped. \( \phi \) and \( \alpha \) are the initial phase and gain of the response, respectively.
To model the transient CFO, we define the overall CFO in terms of two parts: a transient CFO and a steady state CFO. We assume that the steady state CFO, denoted by $\Delta \omega_S$, is constant during the preamble OFDM symbol. Then, the overall CFO can be written as $\Delta \omega(t) = \Delta \omega_T(t) + \Delta \omega_S$. Thus, the received signal become

$$r(t) = \left( \int_{-\infty}^{+\infty} h(v) s(t-v) dv + n(t) \right) e^{j\int_{0}^{t} \Delta \omega_T(\tau) d\tau} \quad (6)$$

In this paper, we assume that the steady state CFO is estimated and removed from the baseband signal to simplify the estimation of the transient CFO. This assumption is reasonable in practice since a terminal first detects the downlink preamble signal to obtain the time synchronization, the steady state CFO, and other necessary information to access to the wireless network during the initialization stage. The terminal stays in RX state during this stage and does not have to be running in TDD mode.

Let us now consider the received preamble symbol in two halves

$$r_1(t) = \left( \int_{-\infty}^{+\infty} h(v) s(t-v) dv + n_1(t) \right) e^{j\int_{0}^{t} \Delta \omega_T(\tau) d\tau}$$

$$r_2(t) = \left( \int_{-\infty}^{+\infty} h(v) s(t-v) dv + n_2(t) \right) e^{j\int_{t}^{t+T} \Delta \omega_T(\tau) d\tau}$$

The phase difference between the first and second halves is denoted by $\psi(t)$

$$\psi(t) = \zeta(r_1^*(t) r_2(t)) \quad (7)$$

$$= \int_{t}^{t+T} \Delta \omega_T(\tau) d\tau + n_\psi(t)$$

Where $n_\psi(t)$ is the noise term in the phase difference. Note that $\psi(t)$ is just the definite integral of the transient CFO over interval $[t, t+T]$. Then $\psi(t)$ is still an exponentially damped sinusoid with the same damping factor and frequency, but different initial phase and gain, i.e.,

$$\psi(t) = \int_{t}^{t+T} \Delta \omega(\tau) d\tau + n_\psi(t)$$

$$= \alpha_\psi e^{-\zeta \omega t} \sin(\omega_\tau \sqrt{1 - \zeta^2 t} + \phi_\psi) + n_\psi(t) \quad (8)$$

2.3 Subspace Based Estimation of $a$ and $\omega_s$:

Estimating the parameters of damped sinusoid has been extensively. The existing algorithms are, however, rather generic and can be generally put into four categories: direct fitting in time domain using signal samples, direct fitting in frequency domain with DFT based algorithms, covariance based linear prediction, and subspace(SVD) based linear prediction.
Then, the complex roots of the prediction polynomial are found to compute the damping factor and frequency.

\[
\psi = \begin{bmatrix}
\psi(0) & \psi(1) & \ldots & \psi(L-1) \\
\psi(1) & \psi(2) & \ldots & \psi(L) \\
\vdots & \vdots & \ddots & \vdots \\
\psi(N-L-2) & \psi(N-L-1) & \ldots & \psi(N-1)
\end{bmatrix}
\]

The prediction filter order, L, which is also the number of columns of the Hankel matrix, is chosen to optimize the estimation performance in the subspace based linear prediction problems, such as the Kumaresan–Tuft (KT) method, and matrix pencil method. The first order perturbation analysis for the KT method as the subspace based estimator. We use the first order subspace perturbation analysis for the KT method as

\[
Var(\omega_s) \approx \frac{4}{3\sqrt{L(N-L)^2}} \quad 1 \leq L \leq N - L
\]

The proposed subspace based algorithm is described below.

We construct the prediction matrix:

\[
\Psi_r = \begin{bmatrix}
\psi(0) & \psi(r) & \psi(2r) \\
\psi(1) & \psi(r+1) & \psi(2r+1) \\
\vdots & \vdots & \vdots \\
\psi(N-2r-1) & \psi(N-r-1) & \psi(N-1)
\end{bmatrix}
\]

\[
W_r = 
\begin{bmatrix}
\psi(0) & \psi(1) & \psi(2r) \\
\psi(1) & \psi(r+1) & \psi(2r+1) \\
\vdots & \vdots & \vdots \\
\psi(N-2r-1) & \psi(N-r-1) & \psi(N-1)
\end{bmatrix}
\]

Assume that the singular value decomposition (SVD) of \(H_r\) is

\[
H_r = [U_S \ U_0] \begin{bmatrix} \Sigma_S & 0 \\ 0 & \Sigma_0 \end{bmatrix} \begin{bmatrix} \psi_s^H \\ \psi_0^H \end{bmatrix} = U_S \Sigma_S \psi_s^H
\]

2.3 Performance Analysis and Weighted Subspace Fitting: The WSF algorithm is proposed based on the perturbation analysis of the subspace based estimator. We use the first order subspace perturbation analysis for the Hankel-like matrix \(\Psi_r = H_r + W_r\), where \(\Psi_r\) and \(W_r\) are defined.

\[
\Delta \psi_0 \approx -V_s \Sigma_s^{-1} U_s^H \Psi_r \psi_0
\]

A first-order approximation of the perturbation at high SNR is given by

\[
\Delta \psi_0 \approx \frac{|v_{01}|^2}{\rho((i+r-1)/T_s)} + \frac{|v_{02}|^2}{\rho((i+2r-1)/T_s)} + \frac{|v_{00}|^2}{\rho((i-1)/T_s)} + \frac{|v_{01}|^2}{\rho((i+r-1)/T_s)} + \frac{|v_{02}|^2}{\rho((i+2r-1)/T_s)}
\]

Assume \(\psi_0 = [v_{00}, v_{01}, v_{02}]^T\). Defining the instantaneous SNR as \(\rho(nT_s)\), we have

\[
|\Gamma|_{i,i} = \frac{|v_{00}|^2}{\rho((i-1)/T_s)} + \frac{|v_{01}|^2}{\rho((i+r-1)/T_s)} + \frac{|v_{02}|^2}{\rho((i+2r-1)/T_s)}
\]

\[
|\Gamma|_{i,i+r} = \frac{v_{00}^* v_{01}}{\rho((i+r-1)/T_s)} + \frac{v_{01}^* v_{02}}{\rho((i+2r-1)/T_s)}
\]

\[
|\Gamma|_{i,i+r} = \frac{v_{00}^* v_{02}}{\rho((i+2r-1)/T_s)}
\]
We propose a weighted subspace fitting method in order to reduce the MSE of the estimated $v_0$. The weighted subspace fitting can be modeled as

$$\Psi_{RD} = D \Psi_r = DH_r + DW_r$$  \hspace{1cm} (17)

### 2.4 Selection of Time Lag $r$:
In constructing the matrix $H_r$, it is important to select the time lag $r$. In this section, we investigate the best strategy to select $r$. Let us recall that the mean square error of $\Delta v_D$ in

$$E[\|\Delta v_D\|^2] = Tr((U_s^H R^{-1} U_s)^{-1} \Sigma_s^{-2})$$

$$\leq Tr((U_s^H R^{-1} U_s)^{-1}) Tr(\Sigma_s^{-2})$$

$$\leq \sum_{n=0}^{N-1} \frac{1}{\rho|nT_s|} \frac{(\sigma_1^2 + \sigma_2^2)}{(\sigma_1^2 \sigma_2^2)}$$  \hspace{1cm} (18)

The sample covariance matrix of noise free matrix $H_r$ is asymptotically equivalent to

$$\lim_{N \to \infty} \frac{1}{N-2r} H_r^H H_r = F_3^{(r)} F_3^{(r)^H}$$  \hspace{1cm} (19)

Where $F_3^{(r)}$ is a Vandermonde matrix defined as

$$F_k^{(l)} = \begin{bmatrix}
1 & e^{i(a+j \omega l)} & e^{i(a-j \omega l)} \\
1 & e^{i(a+j \omega (k-1))} & e^{i(a-j \omega (k-1))} \\
\vdots & \ddots & \ddots \\
1 & e^{i(a+j \omega (k-1) - 1))} & e^{i(a-j \omega (k-1) - 1))}
\end{bmatrix}$$  \hspace{1cm} (20)

### 2.5 Estimation of the Initial Phase and Gain
Observing that the noise of the phase difference vector is not white. We can use a whitening solution as follows. Assume the covariance matrix of $\psi$ is $R$, and the positive definite matrix $R^{-1}$ can be factored as

$$R^{-1} = S^H S$$  \hspace{1cm} (21)

Then $S$ acts as a prewhitening matrix since the covariance matrix of $S \psi$ is an identity matrix. The estimation of $r$ with prewhitened least squares method is

$$\hat{r} = \left((S F_N^{(1)})^H S F_N^{(1)} \right)^{-1} (S F_N^{(1)})^H S \psi$$  \hspace{1cm} (22)

### 3. EXPERIMENTAL RESULTS

![Figure: comparison between original And estimated signals](image)

![Figure: Comparison of the MSE of the estimated damping](image)
CONCLUSION:

Here in this paper we are analyzed a unique problem in the wireless communication system i.e. CFO (carrier frequency offset). It was mainly observed in switching systems of transmitter and receiver. To reduce this problem we are proposed a algorithm based on the subspace decomposition of the Hankel-like matrix. And to improve the antenna accuracy a weighted subspace fitting algorithm is proposed in both numerical simulations and experimental results. The performance analysis is verified from the tested collected samples.

REFERENCES


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Working as Assistant professor in Malla Reddy Institute of Technology & Science, having 5 Years of teaching experience